**Differentiation**

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## Tangent and Derivative at a Point

Tangent to the graph of a function:

Slope of a secant line: average rate of change

Instantaneous rate of changed, slope at a single point.

Find the equation for the tangent to the curve at a given point:

1. at
2. at
3. at

Find the equation of all lines with slope that are tangents to the curve .

and

Slope of a tangent at , for

Instantaneous rate of change at

This rate of change at has a special name. It is called the derivative at for and defined as , provided a limit exists.

What is the rate of change of the area of a circle () with respect to the radius where ?

If , then find

1. Average rate of change from to
2. Instantaneous rate of change at

An object is dropped from the top of a high tower. Its height above the ground after seconds is . How fast is it falling after it is dropped?

### Horizontal Tangent Line

When the tangent line is horizontal, the slope of the tangent line is .

At what point for the function does a horizontal tangent line exist?

### Vertical Tangent Line

When the tangent line is vertical, the slope of the tangent line is undefined.

## Derivative as a Function

Derivative at

Generally, for the whole domain, , provided that a limit exists.

is the derivative of with respect to .

Find the derivative:

Left Hand Derivative (L.H.D.)

approaches from the left side

Right Hand Derivative (R.H.D.)

approaches from the right side

If L.H.D. R.H.D., function is differentiable at that point.

Functions are not differentiable at corners, cusps, vertical tangency, discontinuous points, rapidly oscillating points (line at ).

Differentiability implies continuity.

Theorem: If is differentiable at , then is continuous at .

## Differentiation Rules

1.

2.

3.

4.

5.

So, there is no turning point.

and

and

## Derivatives as a Rate of Change

Rate of Change – instantaneous rate of change

Motion along a line: displacement, velocity, speed, acceleration, jerk

Displacement: articular position at any time ,

Velocity: Rate of change of displacement , ,

Speed: Absolute value of velocity

Acceleration: Rate of change of velocity

Jerk: Rate of change of acceleration

* Particle moves forward if
* Particle moves backward if
* Speeding up – and same sign
* Slowing down – and opposite signs

Displacement average

Speed

Displacement

Average Velocity

(N.A.)

and

and

## Trigonometric Derivatives

Find all points on the curve , , where the tangent line is parallel to the line .

Here, .

So, the slope of a tangent line of the curve is

Since the tangent line is parallel to the line ,

If , is there any tangent line?

An object attached to a string is pulled down from its rest position and released. If the time of oscillation is , then develop a model that relates displacement to time.

## The Chain Rule

## Implicit Differentiation

At ,

At ,

Equation of tangent:

Equation of normal:

Verify that the following pairs of curves meet orthogonally:

## Related Rules

Rate of change of Area

Rate of change of radius

Assume that oil spilled from a ruptured ranker spreads in a circular pattern with radius increasing at a constant rate of . How fast is the area of the spill increasing when the radius is ?

A rectangle has sides and , with diagonal . Find the rate of change in at the moment when and , given that is increasing at and is decreasing at .

, ,

A ladder, long, leaning against a wall begins to slide down. At the moment when its horizontal distance from the base of the wall is , the rate of change in the horizontal distance is . Find the rate of change in the vertical distance of the ladder from the floor, and the rate of change in the area under the ladder.

,

A pipe with radius and height is losing water at . Find the rate at which the heigh of water in the pipe is decreasing when the height has decreased by .